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Perturbation methods and numerical methods were employed to study four problem areas in fluid dynamics. The areas and the progress were: *1. Viscous vortical flows* - We showed how to combine the asymptotic theory and experiments to study slender vortex filaments and how to specify the numerical parameters needed for the vortex element method to predict correctly the motion of slender filament(s) in space. We presented formulas relating a rotational flow outside a sphere to the rotational flow in space and formulas defining the far-field sound. These formulas were used to study the interaction of a vortex filament with a sphere. *2. Shock wave interactions* - A canonical nonlinear elliptic problem was formulated and used to correct the defect of linear theory near a singular ray where a weak shock interacts with a diffracted wave. A similar canonical problem was formulated to solve the interaction of a weak expansion wave with a diffracted wave. *3. Wave propagation* - Rules were formulated for determining the multiplicity of acoustic signals and the retarded times for media moving with unsteady speed ranging from subsonic to supersonic. In the analysis of structural/acoustic interactions, the solution for the panel oscillation was uncoupled from the acoustic field by the formulation of the *on surface* conditions taking into account the acoustic effect. *4. Free boundary problems* - We obtained solutions for the formation of a drop after the breaking up of symmetric slender jets or thin sheets.

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FINAL REPORT

For the Period: November 1, 1992 - December 31, 1995.

PERTURBATION PROBLEMS IN FLUID DYNAMICS

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TABLE OF CONTENTS

ABSTRACT

Perturbation methods and numerical methods were employed to study four problem areas in fluid dynamics. The areas and the progress were: *1. Viscous vortical flows* - We showed how to combine the asymptotic theory and experiments to study slender vortex filaments and how to specify the numerical parameters needed for the vortex element method to predict correctly the motion of slender filament(s) in space. We presented formulas relating a rotational flow outside a sphere to the rotational flow in space and formulas defining the far-field sound. These formulas were used to study the interaction of a vortex filament with a sphere. *2. Shock wave interactions* - A canonical nonlinear elliptic problem was formulated and used to correct the defect of linear theory near a singular ray where a weak shock interacts with a diffracted wave. A similar canonical problem was formulated to solve the interaction of a weak expansion wave with a diffracted wave. *3. Wave propagation and acoustic/structure interaction* - Rules were formulated for determining the multiplicity of acoustic signals and the retarded times for media moving with unsteady speed ranging from subsonic to supersonic. In the analysis of acoustic/structure interactions, the solution for the panel oscillation was uncoupled from the acoustic field by the formulation of the on surface conditions taking into account the acoustic effect. *4. Free boundary problems* - We obtained solutions for the formation of a drop after the break up of symmetric slender jets or thin sheets.

1. INTRODUCTION

We have used perturbation and numerical methods in our investigations of fluid dynamics problems for many years. Our investigations are centered on four problem areas. They are 1) viscous vortical flows, 2) shock wave interaction 3) wave propagation and acoustic/structure interaction, and 4) free boundary problems. We shall describe briefly our earlier research activities leading to our current investigations in these areas.

1.1 Viscous Vortical Flows The research in this area has been carried out along three lines, a) slender vortex filements, b) distributed vortical fields and c) far-field sound induced by vortical flows. A comprehensive review of the research prior to 1990 was presented by Ting and Klein [1].*

The motion and diffusion of slender vortex filaments was analyzed by the method of matched asymptotics in 1965 by Ting and Tung [2] for two dimensional problems and extended to axi-symmetric problems in 1967 and finally to filament(s) in space with both large swirling and axial flows in the core structure by Callegari and Ting in 1978 [3]. The analysis defines the temporal evolution of the core structure and the velocity of the filament, establishing the dependence of the velocity on the vortical core structure. These results were needed since the velocity was undefined in the inviscid theory and was unbounded in the limit of zero core size (see for example Lamb [4]). The analysis of Callegari and Ting [3], which assumes that the core structure does not vary in the axial direction, i. e., along the filament, was extended by Klein and Ting in 1992 [5] to allow for axial variation of the core structure. The axial variation is essential in the modeling of the vortex breakdown phenomenon, which is an important problem for high speed wings at finite angle of attack (see for example [6]). The extended analysis was employed in the numerical modeling of vortex breakdown and the results were presented by Schmitz et al [7] at the 6th Intern. Symp. Comput. Fluid Dynamics, Lake Tahoe, NV, Sept. 4-8, 1995. Klein and Ting showed in 1995 [8] how to combine the asymptotic theory and experiments to study slender vortex filaments and how to specify the numerical parameters needed for the vortex element method to predict correctly the motion of slender filament(s) in space. Using this theory [8], a vortex element code was developed and numerical examples were obtained by Klein, Knio and Ting. They presented the essence of the investigation [9] at the ICIAM95 meeting in Hamburg, in July 1995. A manuscript [10] based upon this talk was submitted recently to Physics of Fluids.

To study the evolution of a distributed vorticity field, i. e., not confined in a slender filament(s), we need numerical solutions of Navier-Stokes (N-S) equations. To develop an efficient numerical scheme for the solution in a finite computational domain, we need boundary data simulating the solution in an unbounded

* The number inside square brackets denotes the reference number listed in §4

domain. By making use of the invariants for the moments of vorticity of Truesdell and Moreau (see for example [11]), formulas for the generation of the appropriate boundary data were presented by Ting in 1983 [12]. These formulas were employed to carry out numerical simulations of the interaction and merging of filaments in space (see for example [13, 14]). Numerical studies of the evolution of a vorticity field with total strength zero were carried out by Ting and Bauer in 1993 [15]. Because of the integral invariants, the vorticity field behaves as a doublet of constant strength in the far field. Numerical studies in [15] deal with the motion and merging of viscous vortices in two- and three-dimensional space and their long time behavior.

The far-field sound induced by a vorticity field in space was derived systematically by the method of matched asymptotic expansion by Ting and Miksis in 1990 [16]. They show that the leading acoustic field is composed of quadrupoles and there strengths are given by the time-derivative of the second moments of vorticity. To study the interaction of a vortex filament with a rigid body, say a sphere, Knio and Ting in 1995 [17] derived formulas relating a rotational flow outside a sphere to the rotational flow in space and formulas defining the far-field sound. Due to the presence of the sphere, the leading acoustic field has dipoles in addition to the quadrupoles. These formulas were used to study the interaction of the filament with a rigid sphere and compute the far-field sound. The results differ qualitatively and quantitatively from those without the sphere. These investigations by Knio, Klein and Ting [18] will be presented in the 19th ICTAM meeting in Kyoto, August 1996.

The research reported in [7] to [10], [15], [17] and [18], supported by this grant, will be elaborated in §**2.1**.

1.2 Shock Wave interaction We analyze semi-linear waves and construct uniformly valid second order corrections to linear solutions with weak shocks and/or expansion fans.

The singularities produced by the nonlinear interaction of progressing waves in semi-linear wave equations in two-dimensional space were studied by Rauch and Reed in 1982 [19] and by Melrose and Ritter in 1985 [20]. Studies of the locations and types of singularities of semilinear waves in three and higher dimensional space were initiated under a previous AFOSR grant, number AFOSR-90-0022 and was completed under the current grant in collaboration with Prof. J. B. Keller at Stanford [21]. The analysis and the conclusions will be elaborated in §**2.2**.

Two linear theories of shock diffraction problems were developed: one for the diffraction of a shock of finite strength by a thin or slender body and one for the diffraction of a weak shock by a finite body. Conical solutions for the diffraction of a shock of finite strength by thin wedges were presented by Lighthill in 1949 [22]. Conical solutions for the diffraction of weak shocks by wedges and corners was obtained by Keller and Blank in 1951 [23]. Explicit solutions for the diffraction of a shock of finite strength by a thin symmetric airfoil of infinite span were obtained by Ting and Ludloff in 1951 [24], a generalization of Lighthill's conical solution [22]. The extension to three-dimensional flows was done by Ting and Gunzburger in 1970 [25] to study the diffraction of a shock by a moving thin wing at a small angle of attack.

The diffraction of a weak shock by structures in a two-dimensional space was studied by Ting in 1953 [26] as an extension of the conical solutions of Keller and Blank [23]. The analysis in [26] was later applied to study the wing-body interactions in supersonic flight in 1957 [27] and extended to study the diffraction and reflection of sonic booms by structures in a three-dimensional space in 1968 [28] and in 1977 [29].

To improve the linear theories, the regular perturbation method fails near the sonic circle (or Mach cone) where the inhomogeneous terms in the second order equation become singular, due to the singularities of the derivatives of the leading order solution. Uniformly valid solutions near the sonic circle were constructed by the Lighthill's technique (1949) [30], by boundary layer method [31] and by the method nonlinear geometric acoustics [32, 33]. All three methods breaks down near a critical or singular ray, where a weak shock is tangent to the sonic circle. Past attempts to render the solution uniformly valid near a singular ray ended in the formulation of transonic equations connecting supersonic to subsonic regions. In collaboration with Prof. J. B. Keller, we formulated recently [34] a canonical nonlinear *elliptic* problem to correct the defect of linear theory near a singular ray where a weak shock interacts with a diffracted wave. A similar canonical problem was formulated to solve the interaction of a weak expansion wave with a diffracted wave. The formulation and the physical justification will be outlined in §**2.2**.

1.3 Wave Propagation and Acoustic/Structure Interaction The scattering of weak pulses or acoustic waves by scatterers or inhomogeneous media has been an important practical problem. Asymptotic methods have been employed to study sound propagation through jets by Ting in 1980 [35] and the jump conditions

across thin bubbly layers by Ng and Ting in 1986 [36]. In general, numerical solutions are needed to simulate scattering problems. To carry out numerical solutions in a finite computational domain, exact boundary conditions simulating the scattering problem in an unbounded domain were presented by Ting and Miksis in 1986 [37]. The next objective is to find conditions under which the acoustic field can be uncoupled from the dynamics of the interface separating two media. In the analysis of the scattering of a pulse by a membrane, it was found by Kriegsmann and Scandrett in 1989 [38] that the solution of the acoustic field can be uncoupled from the solution of the membrane oscillation when the second order radiation conditions are imposed on the membrane surface. The second order terms are needed when the incident wave is nearly in resonance with a natural frequency of the membrane and the accuracy of the *on surface* conditions are established by comparison with the numerical solution of the coupled problem. Using asymptotic analysis, higher order *on surface* conditions were derived systematically by Miksis and Ting in 1989 [39] for the two dimensional problem when the acoustic speed is much smaller than the surface wave speed. We extended the systematic derivation of the *on surface conditions* to the three dimensional problem of acoustic/panel interaction [40]. The generalization of the analysis to an acoustic field moving at a constant speed or at a constant acceleration was carried out in 1994 [41]. This is an important model problem which simulates the excitation of nonlinear panel oscillation by incident waves (e. g., jet noise) and the transmission of sound through the fuselage (see e. g., [42]). The accuracy of the panel oscillation using the on-surface conditions to uncouple from the acoustic field, with acoustic speed much smaller that the surface wave speed, is verified by comparison with the numerical solution of the coupled system for acoustic/panel oscillation. The comparison was presented in 1994 IMA workshop *Computational Wave Propagation* [43]. In the experimental investigations on the jet noise and panel interactions by Dr. Lucio Maestrello at NASA Langley Research center, he observed that the characteristics of the panel oscillation and the scattered field change drastically when the jet is unsteady, accelerating or decelerating. This fact prompted the theoretical study on acoustic/panel oscillation for a medium accelerating at a constant rate from subsonic to supersonic speed. This study, reported in the second part of [41], in turn produced clues for the analysis of wave propagation in a medium moving at an unsteady speed $U(t)$ along a fixed direction $\hat{\imath}$. The analysis was carried out by Bauer, Maestrello and Ting in 1995 [44] in which rules were formulated for determining the multiplicity of acoustic signals and the retarded times for media moving with unsteady speed ranging from subsonic to supersonic.

The research reported in [40], [41], [43] and [44], supported under this grant will be elaborated in §**2.3**.

1.4 Free Surface Problem The motion of an interface separating two media is a free surface problem. In general the problem is nonlinear unless the interface is only slightly disturbed. For an interface separating a liquid from a gas phase, e. g., the water wave, the inertia terms in the gas phase can be neglected and hence the pressure on the interface from the liquid phase can be considered as constant. Analytical solutions were obtained for free boundary problems in the merging of isolated vortices and in the corresponding problems in two-dimensional elasticity by Ting in 1977 [45]. An asymptotic method was applied successfully to free surface problems, namely, the planing of a flat plate at high Froude number in 1974 by Ting and Keller [46], the buckling of a viscous column by Buckmaster, Nachman and Ting in 1975 [47], Buckmaster and Nachman in 1978 [48] and the surface wave induced by an impinging jet by Miksis and Ting in 1983 [49]. The drop formation in the breaking of a jet was studied by Taylor in [50] and by Keller [51]. The asymptotic method was applied by Ting and Keller in 1990 [52] to study the similarity solutions of slender jets and thin sheets with surface tension and the connection to drop formation. The solutions for the formation of a drop after the break up of symmetric slender jets or thin sheets were obtained by Keller, King and Ting in 1995 [53]. The solutions will be described in detail in §**2.4**.

This grant supported the research effort of Lu Ting and Frances Bauer at CIMS. The investigations in problem area 1) were carried out with the collaborations of Prof. Egon Krause and Dr. Rupert Klein, RWTH Aachen and Prof. Omar Knio, Johns Hopkins University; in areas 2) and 4), with Prof. Joseph B. Keller, Stanford University; and in area 3) and with Prof. Michael J. Miksis, Northwestern University and Dr. Lucio Maestrello, NASA Langley Research center. A list of publications and presentations of research supported by this grant is given in §**3**.

2. HIGHLIGHTS OF THE INVESTIGATIONS

In §**1**, we described briefly our earier research activities leading to our current investigations in four problem areas: 1) viscous vortical flows, 2) shock wave interaction 3) wave propagation and acoustic/structure interaction, and 4) free boundary problems. Now we elaborate on the objectives and acomplishments of our current research in these four areas in subsections §**2.1** to §**2.4** respectively.

2.1 Viscous Vortical Flows

We present the highlights of our investigations on slender vortex filaments in §**2.1a)**, on distributed vortical fields in §**2.1b)**, and on far-field sound induced by vortical fields in §**2.1c**.

2.1a) In collaboration with Rupert Klein, Institut für Technische Mechanik, RWTH Aachen, we address the questions of how to explain the asymptotic theory of slender vortex filaments to experimental fluid dynamicists, so that they can use it to design and calibrate the experiments and to find the corresponding theoretical solution to compare with the experimental data. The answers to these questions are presented in a paper entitled "Theoretical and Experimental Studies of Slender Vortex Filaments" in Appl. Math. Letters. 1995 [8] The abstract of the paper is

Abstract– We show how theoretical analysis and experimental investigation complement each other in the study of the dynamics of slender vortex filaments. The asymptotic solution of a single filament requires the initial data of the shape of the centerline and the core structure. The latter is usually not available from experimental data. When the viscous diffusion time t_d of the core is of the order of the duration t^* of the experiment, we can define the contribution of the core structure to the filament velocity by a finite number of parameters, which can then be determined by comparing the theoretical predictions of the motion of the centerline with the experimental data. These parameters in turn calibrate the single vortex generator in the experimental set up by defining the moments of the initial core structure. In the inviscid limit, where $t^* \ll t_d$, we derive the formulas for the contributions of the the core structure to the filament velocity and show that they depend only on two initial constants. The analytical solution can then be used to model the interaction of filaments with or without boundaries or interfaces, and compare with the experimental data.

This paper forms the basis of an invited lecture, "Recent Development on the Dynamics of Slender Vortex Filaments and their Vortical Core Structure" by L. Ting and R. Klein in *Vortex Flows in Aeronautics*, International Colloquium of the SFB 25, RWTH Aachen, October 12–14, 1994. Extension of the analysis and numerical examples are being carried out by R. Klein, O.M. Knio and L. Ting. The results were presented in an invited paper [9], "Accurate numerical simulation of stretched high Reynolds number slender vortices in space", in a mini-symposium, *On Vortical Flows*, in ICIAM95, Hamburg, July 3–7, 1995.

An invited talk, entitled "Recent Advances in the Analysis and Numerical Computation of Slender Vortex Filaments" by M. Schmitz, RWTH Aachen, O.M. Knio, L. Ting, and R. Klein was presented at the Sixth International Symposium on Computational Fluid Dynamics (ISCFD), September 4-8, Lake Tahoe, NV. The extended abstract [7] is:

Extended Abstract– The systematic asymptotic analysis of slender vortices in three dimensions has recently been extended in several directions. This lecture will discuss two of these advances regarding:

1) The influence of axial variation of the core structure on the motion and internal structure of a filament.
2) The combination of asymptotic analysis and modern vortex element numerical techniques for highly accurate computations of slender vortex motion – including the effects of vortex stretching as well as of viscous diffusion of vorticity.

Most previous analyses of the structure of slender vortex filaments have used the tacit assumption that the leading order vortex core structure does not vary along the filament centerline. A detailed asymptotic analysis reveals that the vortex core structure of a three-dimensional vortex filament must satisfy two constraints that have been known from the theory of axisymmetric slender

vortices for a long time: The total head on and the circulation around tubes of constant axial mass flux are constant along the filament. These constraints naturally lead to structure equations that are equivalent to the well-known axisymmetric slender vortex equations of Hall and Benjamin. From earlier studies of these equations it is known that for suitable radial distributions of total head and circulation and under certain outer boundary conditions a critical state can develop. This critical state separates sections of the vortex filament on which small amplitude perturbations can and cannot travel upstream against the prevailing axial flow (subcritical / supercritical transition). Benjamin has associated this sub- to supercritical transition with the vortex breakdown phenomenon.

We provide further insight into the solution behavior near the critical state and show that the details of the transition are crucially influenced by the outer boundary conditions imposed on the vortex core structure. If the vortex is embedded in a tube with rigid walls and diameter comparable to the vortex core size, then near the critical state a rapid axial variation occurs on axial length scales comparable to the core diameter. In this case, a consistent description of the transition must resort to solving the full axisymmetric inviscid flow equations. A simplifying slenderness assumption is not appropriate. If the location of the outer boundary is, however, far away from the core (measured on the core size scale), which is the typical situation in applications, then the flow in the transition region remains slender. For two regimes of characteristic diameters of the outer boundary, two new sets of simplified structure equations can be derived whose complexity is in between that of the full axisymmetric flow equations and Benjamin's slender vortex equations. We discuss numerical solutions of these new asymptotic equations with emphasis on the transition through the critical state. We will support the theory by comparison with direct numerical simulations.

These efforts in analyzing the vortex core structure should be viewed in close connection with the motion of a vortex in three dimensions. It is well known that the detailed core structure influences nontrivially the local self-induction, so that axial variations of the core structure may lead to geometrical distortions of the vortex geometry that would not occur for a filament with axially constant structure. These considerations are particularly important in connection with the phenomenon of vortex breakdown: If there is a weak overall vortex curvature and locally an axisymmetric breakdown with a super- to subcritical transition is incipient, then the strong axial core structure variation will lead to a strong bending of the vortex right at the point of breakdown – due to the associated change of the self-induction. Hence, a direct connection between axisymmetric and spiral modes of vortex breakdown appears.

An accurate description of the vortex filament motion requires sophisticated numerical tools that overcome the extreme demands imposed by highly concentrated vorticity distributions. We describe in the second part of the lecture a recent combination of asymptotic analysis and vortex element numerical techniques that has allowed us to design a highly accurate numerical method for simulating the vortex filament dynamics taking into account appropriately the influence of core structure variations on the filament motion. The idea is to compare the asymptotic structure of the vortex element scheme in the thin-tube approximation with a detailed asymptotic description of the vorticity structure of a physical vortex. This comparison reveals the major source of numerical errors and guides the design of a suitable correction of the scheme. One central improvement achieved in this fashion is that the numerical results become essentially independent of the choice of the numerical vorticity smoothing function – a point of major critique against earlier thin-tube vortex element computations.

We present applications of the scheme to problems of interacting vortex rings, and exhibit the influences of vortex stretching as well as of vorticity diffusion in the core.

A manuscript describing the essence of our talk at ICIAM95 [9] and additional numerical results was prepared and submitted to Physics of Fluids, Part A. The title is "Representation of Core Dynamics in Slender Filament Simulation" [10] and its abstract is

Abstract– The numerical description of slender vortex motion faces several major obstacles: (i) The stiffness induced by the rapid rotatory motion in the vortex core, where the peak velocities are an order of magnitude larger than the filament velocity. In a vorticity-velocity formulation, this stiffness is reflected by the singular behavior of the Biot-Savart line integral as one approaches the

vortex centerline. Regularization occurs physically by viscous smoothing of the vorticity. (ii) The vortex core vorticity distribution has a crucial influence on the vortex filament motion. Thus an accurate description of the core structure evolution due to vortex stretching and vorticity diffusion is necessary. We propose a numerical scheme that allows accurate description of the effects of axial flow in the core, viscosity and vortex stretching on slender vortex filament motion. The approach is based on incorporating the detailed asymptotic analysis of the core structure evolution by Callergari and Ting (1978) and Klein and Ting (1995) for stretched viscous slender vortices into the improved thin-tube vortex element schemes of Klein and Knio (J. Fluid Mech. **284**, 275-321, 1995). The resulting schemes overcome the difficulties mentioned above except for the issue of temporal stiffness, which we leave for future work.

2.1b) We study the evolution or merging of an initial vorticity distribution concentrated in a few spots or slender tubular regions in two- or three-dimensional space to a distributed vorticity field. We say that the flow field is induced by viscous vortices since the viscous effects are needed to account for the diffusion of the slender core structure. As the core size increases and becomes comparable to the background length scale or the size of a vortex filament we have the merging of vortices. For an exponentially decaying vorticity field, the total strength is zero and hence the far field behaves as a doublet. Thus it is of interest to study the motion and evolution of a viscous doublet, which is a vortical field with total strength zero. Note that the total strength of a two- or three-dimensional viscous doublet is defined by the first moments of vorticity and is a time invariant vector **E**. In the inviscid theory, *stationary* doublets were employed as singularities inside a body to simulate the flow field around the body. The velocity of a *free inviscid* doublet is not defined. As we see from [15], the velocity of a free viscous doublet depends on the vorticity distribution. In collaboration with Dr. F. Bauer, we studied the dynamics of viscous doublets by asymptotic and numerical methods [15] and showed the dependence of the velocity of a free viscous doublet depends on the vorticity distribution. The results are presented in a paper entitled "Viscous vortices in two- and three -dimensional space", in *Comp. and Fliuds*, Vol. 22, 565-588, 1993 [15]. The abstract of this paper is:

Abstract– Recent developments for the interaction, diffusion and merging of incompressible viscous vortices in two and three dimensional space are presented. We study the motion and evolution of vorticity fields $\boldsymbol{\Omega}(t, \mathbf{x})$ with total strength zero, $< \boldsymbol{\Omega} >= 0$, which is true for a three-dimensional vorticity field decaying exponentially in $|\mathbf{x}|$ and for a two-dimensional case if $< \boldsymbol{\Omega} >= 0$ initially. We called this type of vorticity field a viscous doublet. The strength of the doublet is defined by the time invariant first moments of vorticity. The velocity of the center of the doublet is defined in terms of the second moments of vorticity for two- and three-dimensional problems. We show that the long time behavior of the trajectory of a doublet center in three-dimensional space is different from that in two-dimensional space. Examples are the motion and merging of a slender vortex ring and a two-dimensional vortex pair. For the intersection of two slender filaments (the merging of two segments in a short time interval), a simple model to simulate the merging process and criteria for the reconnection of the filaments after the merging stage are proposed.

2.1c) To study the interaction of a vortex filament with a rigid body, say a sphere, Knio and Ting in 1995 [17] derived formulas relating a rotational flow outside a sphere to the rotational flow in space and formulas defining the far-field sound. Due to the presence of the sphere, the leading acoustic field has dipoles in addition to the quadrupoles. The manuscript [17] describing the analysis and the formulas was submitted to SIAM J. Appl. Math. Its title is "Vortical flow outside a sphere and sound generation" and its abstract is:

Abstract– Formulas are presented for an incompressible inviscid velocity field **V** with a vorticity field $\boldsymbol{\Omega}$ outside of a rigid sphere and for the far-field sound generation. The velocity **V** is expressed as the sum of an *image* velocity $\mathbf{v}^*$ and a known velocity $\mathbf{v}$ in $\Re^3$, which is induced by the same vorticity field $\boldsymbol{\Omega}$ outside the sphere and the extension, $\boldsymbol{\Omega} = 0$ inside. We derive formulas for the *image* velocity, $\mathbf{v}^*$, and the corresponding *image* potential, Φ^*, which in turn yields the far-field sound. These formulas are applied to define the velocity of a slender vortex filament in the presence of a rigid sphere and the associated far-field sound.

These formulas were used to study the interaction of the filament with a rigid sphere and compute the far-field sound. The results differ qualitatively and quantitatively from those without the sphere. These

investigations by Knio, Klein and Ting [18] will be presented in the mini-symposium on *Aero- and Hydroacoustics*, 19th Intern. Congress of Theor. and Appl. Mech., in Kyoto, August 1997. The title of the talk is "Interaction of a slender vortex filament with a rigid sphere: dynamics and far-field noise". Its abstract is

Abstract– Interactions between a slender vortex filament and a stationary sphere are analyzed using a vortex element scheme which tracks the motion of the filament centerline in a Lagrangian reference frame. The filament velocity is expressed as the sum of a self-induced velocity and potential velocity due to the presence of the sphere. The self-induced velocity is estimated numerically using a line Biot-Savart integral which is properly desingularized so as to reflect the correct asymptotic behavior of the core vorticity distribution under the influence of stretching and viscous diffusion. Meanwhile, the potential velocity is evaluated from a recently derived formula, which expresses the potential velocity as a line integral along the image of the filament centerline in the sphere with regular weight functions. From the far-field behavior of the self-induced and the potential velocities, formulas for the acoustic field are obtained. It is shown that the interaction between the slender vortex filament and the sphere generates dipoles and quadrupoles, whose strength and orientations are completely determined by the time evolution of the weighted first and second moments of vorticity. The model is applied to compute the far-field sound generated by the passage of a slender vortex ring over the sphere. Both coaxial and non-coaxial passage events are considered in the analysis, which also examines the effects of initial core size and asymmetric perturbations.

2.2 Shock Wave Interaction

In collaboration with Prof. J. B. Keller, Stanford University, we studied shock wave interaction in unsteady and/or supersonic flows. We identified the singularities in the interaction of weak shocks and the results are presented in a paper entitled "Singularities in Semilinear Waves" *Comm. Pure Appl. Math.* Vol. XLVI, 341-352 , 1993 [21]. The highlights of this paper are:

New singularities of solutions of semilinear hyperbolic partial differential equations, or systems, may be produced when previously existing singularities collide. Our goal is to exhibit the complicated structure of these new singularities for the semilinear wave equation,

$$(\partial_t^2 - \Delta)u = f(u) \ ,$$

in N dimensions, $N \geq 2$ with particular attention to $N = 3$. We assume that for $t < 0$ the solution u has identical jump discontinuities on $N+1$ characteristic hyperplanes which are traveling toward the origin and which meet there at $t = 0$. For $t > 0$, these hyperplanes are traveling away from the origin carrying their discontinuities. Inside the expanding simplex bounded by these $N+1$ hyperplanes there are a number of new singularities. For $N = 2$, Rauch and Reed (1982) [19] gave an example of an $f(u)$ for which there is just one new singularity. It is a 5/2 power singularity on the circle inscribed in the triangle bounded by the three lines of jump discontinuities. Then Melrose and Ritter (1985) [20] showed that for any smooth $f(u)$ this circle is the only possible locus of new singularities.

We show that for $N = 2$ in general there is a 5/2 power singularity on this circle. The solution inside the circle is given by a 2-D conical solution.

For $N = 3$ the four planes bearing the initial singularities bound a tetrahedron which shrinks to a point when they collide, and then expands. We shall show that in general the third derivative of u has a logarithmic singularity on the sphere inscribed in the expanding tetrahedron. In addition there are stronger (5/2 power) singularities on the four circular cones from the four vertices tangent to the inscribed sphere. We exhibit special $f(u)$ for which these are all the new singularities, and others for which there are no new singularities at all.

For any N and general $f(u)$, the singularities of the solutions on the sphere inscribed in the expanding simplex, as well as on certain other conical hypersurfaces are defined.

Our analysis is based on a procedure for constructing the solution explicitly. This construction is possible because, for the function $f(u)$ which we consider, the semi-linear equation (1.1) becomes linear inside the simplex for a finite time as in Rauch and Reed (1982) [19]. For example for $N = 3$, the solution inside the sphere inscribed in the expanding tetrahedron is given by a 3-D conical

solution. The solutions inside the four circular cones from the four vertices tangent to the inscribed sphere are 2-D conical solutions and provide the boundary data for the 3-D conical solution inside the sphere.

Since the type of singularity becomes weaker as N increases, we studied the singularities in shock wave interaction in unsteady two-dimensional flows or the corresponding steady three-dimensional supersonic flows. Linearized theory has been used to study steady supersonic flow over thin wings at a small angle of attack, and for weak shock diffraction problems. It was noted by Lighthill in 1949 [30] that the second order regular perturbation solution for the diffraction of a pulse, a weak shock, or expansion wave of $O(\epsilon)$, is not valid near a sonic circle in an unsteady flow (or a Mach cone in a steady supersonic flow). Uniformly valid solutions near the sonic circle were constructed by the Lighthill technique in 1949 [30], by boundary layer methods (Zahalak and Myers, in (1974) [31]), and by a systematic expansion procedure for weakly nonlinear geometrical acoustics, (Hunter and Keller, in 1984 [32] and 1987 [33]). These solutions yield $O(\epsilon)$ deformation of the sonic circle and predict $O(\epsilon^2)$ shock or expansion waves. But they all break down near a critical or singular ray where the pulse is tangent to the sonic circle. Past attempts to render the solution uniformly valid near the line of tangency ended in showing that the governing equation is transonic.

We note that the linear theory ignores the $O(\epsilon)$ difference between the sonic circle C_1 in the uniform low pressure region and the circle C_2 in the high pressure region and the differences between the three speeds, the speed of the pulse, the sonic speed ahead, and the speed behind the pulse. Therefore the pulse is tangent to the circle. But in the neighborhood $\mathcal{N}$ of a critical ray, these differences can no longer be ignored because the width and the thickness of $\mathcal{N}$ are $O(\sqrt{\epsilon})$ and $O(\epsilon)$ respectively. Thus the linear theory in $\mathcal{N}$ is not valid. By paying attention to these differences, we succeeded in showing that the governing equation for the local region near the singular ray is nonlinear but elliptic and constructing a canonical solution. The analysis and numerical solutions will be reported in a paper in preparation, entitled *Weak Shock Diffractions*, by Ting and Keller. Its abstract is

Abstract A weak shock is tangent to a weak incident or reflected shock and the flow is singular at the point of tangency, according to linear theory. We correct the defect of linear theory by introducing a canonical solution of a nonlinear elliptic problem in the neighborhood of the point of tangency. We solve a simplified form of this problem to obtain the corrected shock shape and pressure distribution. We also indicate how to use the solution in an iterative method to get a more accurate solution. We formulate and solve a similar canonical problem for the interaction of a weak expansion wave with a weak diffracted wave. Our analysis applies to the self-similar flow fields behind plane pulses diffracted by wedges and corners, and to steady supersonic flows past wings with planar surfaces.

2.3 Wave Propagation and Acoustic/Structure Interaction

In collaboration with Prof. M. J. Miksis, Northwestern University, we study the scattering of an incident wave by a baffled flexible panel. It serves as a model problem for the airframe response to incident waves coming from the jet and the wave transmitted into the airframe.

For a given incident wave above the panel, the acoustic pressure difference across the baffled panel excites panel oscillations which in turn induce the scattered waves. The panel oscillation $\eta(t, x, z)$ is governed by an integro-differential equation which is the differential equation for panel oscillation with the acoustic loading related to the panel oscillation via the standard integral representation of the solution of a simple wave equation in half space. An approximation, which replaces the integral representation for the acoustic loading by the derivatives of η, is called an on-surface condition which reduces the integro-differential equation to a differential equation. Using the ratio of the acoustic wave length to the size of the panel as the small expansion parameter ϵ, we derive by a systematic asymptotic analysis the leading order and second order on-surface conditions. The leading order condition is the well known plane wave approximation. The second order condition is needed when the incident wave is nearly in resonance with the panel oscillation and then the solution is sensitive to acoustic damping with higher order accuracy. The analysis is presented in a paper entitled "Panel Oscillations and Acoustic Waves" published in Appl. Math. Letters [40]. The abstract of the paper is

Abstract The interaction of an acoustic wave and a clamped elastic panel is considered. Using the fact that the ratio of the acoustic wave length in the fluid to the size of the panel is small, we are able to derive by a systematic asymptotic analysis an on-surface boundary condition for this scattering problem.

This analysis and numerical examples were presented as the invited lecture "Structural Acoustics Interactions and on-surface conditions" by M. J. Miksis and L. Ting in IMA *Computational Wave Propagation Workshop*, September 19–23, 1994 University of Minnesota. This lecture will appear in the Proceedings of the Workshop as an IMA Volume, Springer-Verlag. The abstract of this paper [43] is

> **Abstract** The interaction of an acoustic wave and a clamped elastic panel is considered. Here we present numerical results comparing the exact solution found by solving the coupled acoustic wave - elastic panel problem to two decoupling approximations using an on surface boundary condition. These on surface conditions are derived in the limit where the ratio of the acoustic sound speed in the fluid to the surface wave speed in the panel is small.

We are attempting to generalize the above on-surface condition for a single panel with clamped edges on a rigid horizontal plate to a doubly-periodic array of panels with clamped edge. The latter is a more realistic model for a fuselage.

The systematic derivation of the leading and second order on-surface conditions for a medium moving at constant speed, subsonic or supersonic was carried out by Ting. The analysis and results were published in SIAM J. Appl. Math. 1995 [41]. The abstract of the paper is:

> **Abstract** We consider the scattering of an incident wave in moving media by a flexible panel mounted on a large rigid plate. The medium is moving parallel to the panel at a uniform unsteady velocity. Besides the regular reflection, there are scattered waves induced by the panel oscillation which in turn is excited by the acoustic pressure difference across the panel. With the acoustic load on the panel related to its oscillation by an area integral, the panel oscillation, linear or nonlinear, is governed by a closed system of integro-differential equations. This system is applicable to fluid moving parallel to the panel at any Mach number from zero to supersonic. The solution for the panel oscillation can then be used to determine the entire acoustic field, on the incident and the transmitted sides. When the ratio of the acoustic wave length to the surface wave length is small, we use the ratio as an expansion parameter and systematically derive the leading and higher order on-surface conditions relating the pressure on the panel to the derivatives of its oscillation. The higher order relationship is needed when the incident wave is nearly in resonance with a mode of the panel oscillation. The relationships are local and hence the system of integro-differential equations is reduced to a differential equation. The derivation of the approximate on-surface conditions is applicable for linear or nonlinear panel oscillations and for an unsteady uniform flow parallel to the panel at any Mach number.

In the experiments conducted at NASA Langley Research Center by Dr. L. Maestrello, it was found that the characteristics of acoustic/structure interaction, the panel response and the scattered field, are different when the flow is accelerating, at constant speed, or decelerating. A brief discussion of the acoustic field for a flow accelerating or decelerating at a constant rate was presented in the last section of [41].

In collaboration with F. Bauer of Courant Institute and L. Maestrello of NASA Langley Research Center, we analyze acoustic fields in unsteady moving media, with speed varying from subsonic to supersonic or vice versa. The analysis was presented in a paper entitled "Acoustic wave propagation in unsteady moving media" in 1995 [44]. This is an important problem in aeroacoustics. The equations governing the generation of the acoustic source distribution, e. g., the jet noise or waves induced by the oscillation of airframe panels are usually simpler in the coordinate system $\mathbf{x} = (x, y, z)$ moving with the airframe. In the system $\mathbf{x}$, the medium is moving with an unsteady velocity $U(t)\hat{\imath}$. The acoustic field induced by the source distribution is governed by a convective wave equation with variable coefficients involving $U(t)$ and $\dot{U}(t)$. Explicit solution of the convective wave equation or its Green's function in free space is available only for a constant U. An implicit solution can be constructed from the solution in the coordinate system $\bar{\mathbf{x}} = (\bar{x}, y, z)$ with the medium at rest,

since in $\bar{\mathbf{x}}$ the solution obeys the simple wave equation. The solution is implicit because the transformation from the source point $\mathbf{x}'$ to the position $\bar{\mathbf{x}}'$ at the corresponding retarded time τ is implicit and is not necessarily one to one. Signals issued from a source point S in the domain of dependence $\mathcal{D}$ of an observation point P at time t will arrive at point P more than once corresponding to different retarded times, τ, in the interval $[0, t]$. The number of arrivals is called the multiplicity of the point S.

Existing investigations deal with media moving at constant speed, subsonic or supersonic. It is the purpose of this paper to formulate the rules to identify the domains of integration V_i, with a retarded time in each V_i and the rules to count the *multiplicity* for a medium moving at an unsteady velocity $U(t)$. The paper [44] will appear in Journ. Acoust. Soc. Amer. in 1996. The abstract of the paper is

Abstract In the interaction of an acoustic field with a moving airframe we encounter a canonical initial value problem for an acoustic field induced by an unsteady source distribution, $q(t, \mathbf{x})$ with $q \equiv 0$ for $t \leq 0$, in a media moving with a uniform unsteady velocity $U(t)\hat{\imath}$ in the coordinate system $\mathbf{x}$ fixed on the airframe. Signals issued from a source point S in the domain of dependence $\mathcal{D}$ of an observation point P at time t will arrive at point P more than once corresponding to different retarded times, τ, in the interval $[0, t]$. The number of arrivals is called the multiplicity of the point S. The multiplicity equals 1 if the velocity U remains subsonic and can be greater when U becomes supersonic. For an unsteady uniform flow $U(t)\hat{\imath}$, rules are formulated for defining the smallest number of I subdomains V_i of $\mathcal{D}$ with the union of V_i equal to $\mathcal{D}$. Each subdomain has multiplicity 1 and a formula for the corresponding retarded time. The number of subdomains V_i with nonempty intersection is the multiplicity m of the intersection. The multiplicity is at most I. Examples demonstrating these rules are presented for media at accelerating and/or decelerating supersonic speed.

We are in the process of developing a computational code implementing the above theoretical results. We shall then use the experimental data for unsteady supersonic jet noise, obtained by Dr. Maestrello, NASA LaRC, as input data for our code to predict the acoustic field and compare with his experimental measurements.

2.4 Free Surface Problems

In collaboration with J. B. Keller, Stanford University and Andrew King, University of Keele, England, we studied the shape of a drop trailing the broken end of a slender liquid column and that of a cylindrical blob following the edge of a broken sheet of liquid.

When a stream or column of liquid breaks, a new end is formed on each of the two parts of the broken column. Surface tension pulls each new end toward the corresponding part of the column, forming a growing spherical blob of liquid at each end. An analogous cylindrical blob grows along the newly formed edge of a ruptured sheet or film of liquid. To determine the shapes of these blobs, as well as the velocity and pressure distributions within them, assuming potential flow driven by surface tension, we construct expansions valid within the blobs. A scale length δ, proportional to the blob radius, is the small parameter. After finding these blob expansions, we must match them to outer expansions, valid in the rest of the column or sheet. Such outer expansions were found by Ting and Keller [52] under an earlier AFOSR grant. To leading order, we can match them to the blob expansions by using mass and momentum balance. Higher order matching would then require another expansion, valid in the junction region between the blob and the rest of the column or sheet. The analysis and results are presented in a paper entitled "Blob formation" published in Phys. Fluids [53]. The abstract of this paper is:

Abstract When a thread or column of liquid breaks, a growing blob of liquid forms on the broken ends of the thread as surface tension pulls the ends back toward the rest of the thread. Asymptotic expansions of the shape of the blob and of the flow in it are constructed. Similar expansions are found for the cylindrical blob at the edge of a broken film or sheet of liquid. These results supplement previous calculations of the mass and velocity of the blob.

We are in the process of developing the asymptotic solutions for a slender viscous jet with surface tension where the centerline of the jet is a curve in space.

3. PUBLICATIONS AND PRESENTATIONS SUPPORTED BY THIS GRANT

3.1 Publications

(1) Keller, J. B. and Ting, L., "Singularities of Semilinear Waves", Comm. Pure Appl. Math., Vol. XLVI, 341-352, 1993.

(2) Ting, L. and Bauer, F., "Viscous Vortices in Two- and Three-Dimensional Space", J. Compu.& Fluids, Vol. 22, 565-588, 1993.

(2) Ting, L., "On Surface Conditions for Structural Acoustic Interaction in Moving Media", presented in "Workshop on Perturbation Methods in Physical Mathematics", Rensselaer Polytechnic Institute, June 23-26, 1993, SIAM J. Appl. Math., Vol. 55, pp. 369-389, 1995.

(3) Miksis, M. J. and Ting, L., "Panel Oscillations and Acoustic Waves", Appl. Math. Letters, Vol. 8, No. 1, pp. 37-42, Jan. 1995.

(4) Keller, J. B. , King, A. and Ting, L. , "Blob Formation", Phys. Fluids, Vol. 7, pp. 226–228, 1995.

(5) Klein, R. and Ting, L., "Theoretical and Experimental Stidies of Slender Vortex Filaments", Appl. Math. Letters, Vol. 8, No. 2,pp. 45-50, March 1995.

(6) Bauer, F., Meastrello, L. and Ting, L., "Acoustic Wave Propagation in Unsteady Moving Media", ICASE Rep. No.95-39, NASA La RC, May 1995, to appear in J. Acoust. Soc. Amer., Feb. 1996.

(7) Miksis, M. J. and Ting, L., "Structural Acoustic Interactions and On Surface Conditions", to appear in the proceedings of the IMA workshop on Computational Wave Propagation, September 1994, Springer-Verlag.

(8) Knio, O. M. and Ting, L., "Vortical flow outside a sphere and sound generation", submitted to SIAM J. Appl. Math., October 1995.

(9) Klein, R., Knio, O. M. and Ting, L., "Representation of Core Dynamics in Slender Filament Simulations", submitted to Phys. Fluids A, January 1996.

(10) Ting, L. and Keller, J. B., "Weak Shock Diffractions", in preparation.

3.2 Invited Talks

(1) Ting, L., "On Surface Conditions for Structural Acoustic Interaction in Moving Media", presented in "Workshop on Perturbation Methods in Physical Mathematics", Rensselaer Polytechnic Institute, June 23-26, 1993.

(2) Ting, L., "Matched Asymptotic Expansions and its Applications to Numerical Techniques", Lecture series invited by the Institute of Technical Mechanics, RWTH Aachen,
   (1) Thursday July 29, '93, 10:30 to 12:00, "Introduction to Perturbation Methods"
   (2) Thursday July 29, '93, 14:30 to 16:00, "Singular Perturbation and Physical Intuition"
   (3) Friday July 30, '93, 10:30 to 12:00, "Applications of Asymptotics to Numerical Techniques"

(3) Ting, L., "Dynamics of Slender Vortex Filaments" Seminar presented at the Levich Institute, City University of New York, May 10, 1994.

(4) Miksis, M. J. and Ting, L., "Structural Acoustic Interactions and on Surface Conditions", IMA workshop *Computational Wave Propagation*, Univ. Minnesota, Sept. 1994.

(5) Ting, L., "Recent Developments on the Dynamics of Slender Vortex Filaments and Their Vortical Core Structure", invited lecture, International Colloquium on "Vortical Flows in Aeronautics" SFB 25,October 12-14, 1994, RWTH Aachen, Aachen, Germany.

(6) Ting, L., "Dynamics of Slender Vortex Filaments" Applied Math. Seminar, Stanford University, Feb.17, 1995.

(7) Klein, R., Knio, O. and Ting, L., "Accurate Numerical Computation of Stretched, High Reynolds Number Slender Vorteices in Three Dimensional Space", Mini-symposium M-104, *Vortex Dynamics*, ICIAM95, Hamburg, Germany, July 3-7, 1995.

(8) Schmitz, M., Knio, M. O., Ting, L. and Klein, R., "Recent Advances in the Analysis and Numerical Computation of Slender Vortex Filaments", presented at the Sixth Intern. Symp. Comp. Fluid Dynamics, Sept. 4-8, 1995, Lake Tahoe, NV.

(9) Ting, L., "Slender Viscous Vortex Filaments" Mechanical Engineering Seminar, Johns Hopkins University, September 14, 1995.

[10] Knio, O., Klein, R. and Ting, L., "Interaction of a slender vortex filament with a rigid sphere: dynamics and far-field Noise", to be presented in the mini-symposium on *Aero- and hydroacoustics*' 19th International Congress of Theoretical and Applied Mechanics, Kyoto, Japan, August 25-31, 1996.

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